ROTATIONAL BEHAVIOR OF THE MAIN-SEQUENCE STARS AND ITS PLAUSIBLE CONSEQUENCES CONCERNING FORMATION OF PLANETARY SYSTEMS

Su-Shu Huang

Goddard Space Flight Center
National Aeronautics and Space Administration
and
Department of Space Science and Applied Physics
Catholic University of America

29437 29437

ABSTRACT

A phenomenological theory for the behavior of axial rotation of main-sequence stars is proposed here by considering the effect of braking. It does not specify the physical process of braking but does provide a statistical model by which histograms of observed rotational velocities of stars of different spectral types can be explained in terms of the braking strength. It shows that rotation of all stars including those of O and B type has been braked in various degrees during their course of evolution.

As a result of braking, the stellar angular momentum is transferred outwards to the surrounding nebula that may be regarded as the remnant of star formation. This leads plausibly to the formation around the star of a planetary system from the nebula. If the angular momentum is not further dissipated from the nebula, we can estimate the size of the planetary system and its probability of occurrence. It shows that the size of a planetary system around any star is critically dependent upon the mass in the nebula as compared to that of the star itself.

author

I. A THEORY FOR EXPLAINING THE OBSERVED DISTRIBUTION OF STELLAR ROTATIONAL VELOCITIES

Axial rotation of the main-sequence stars stops quite abruptly at about F5 (Struve 1930). It has frequently been attributed to the braking effect of various mechanisms reviewed in a previous paper (Huang and Struve 1960). A more efficient braking mechanism than those previously proposed has recently been suggested by Schatzman (1962) as due to mass loss through stellar magnetic activities.

In the present paper we do not intend to discuss the physical process of how the braking of stellar rotation is actually affected. Rather, we will show phenomenologically that the observed distribution of rotational velocities of stars belonging to different spectral types can indeed be understood in terms of braking strengths.

As the result of a statistical study (Struve 1945; Huang and Struve 1954) it has been concluded that the rotational axes show no preference in their directions in space. This indicates that the rotating stars have not acquired their angular momentum from the galactic rotation of the pre-stellar gaseous medium. Instead, their angular momenta must have been derived from a random process, such as collisions of interstellar clouds, turbulence, etc. (Huang and Struve 1954). Consequently, we may expect that the angular momenta per unit mass, h, of stars are originally distributed according to the Maxwellian law. Finally we would like to call attention here to the fact that this observational consequence has not been fully appreciated. Many theoreticians still take the rotational angular momenta of stars as due ultimately to the galactic rotation of the tenuous pre-stellar medium (e.g. Edgeworth 1946; Hoyle 1960).

For a given spectral type the geometrical radius and the radius of gyration do not vary greatly. Hence, we may take the equatorial rotational velocity, v, to be in direct proportion to h. If we set $x = v/v_m$, where v_m denotes the most probable equatorial rotational velocity, the original distribution of x in a given spectral range should be given by

$$f_0(x) = \frac{4\gamma}{\sqrt{\pi}} x^2 e^{-x^2}, \quad x \leq x_c \qquad (1)$$

according to our assumption. Here x_c is the upper limit of rotational velocities because of rotational instability and γ is a numerical factor to normalize the function $f_0(x)$. Now if we assume that the star rotates as a rigid body, x is obviously related to the angular momentum per unit mass, h, of the star as follows:

$$h = R k^2 v_m x \qquad (2)$$

where R is the geometrical radius and Rk the radius of gyration of the star. We may define $h = h_c$ by equation (2) when $x = x_c$. Then interstellar clouds with $h > h_c$ could very possibly lead to the formation of close binary systems. Since at present we are interested mainly in the behavior of the observed distribution at its small-velocity end which is not significantly affected by the high-velocity tail, we shall set, for simplicity of calculation, $x_c \to \infty$ and consequently $\gamma = 1$.

As a simple model, we propose that the effect of braking is to reduce h of all stars in a given spectral range by a constant amount, h_1. Define $x = x_1$ when $h = h_1$ is substituted in equation (2). It is then obvious that x_1 measures the degree of braking of axial rotation of the stars within a spectral type, namely, the stronger the braking mechanism, the greater the value of x_1. In this way we are able to obtain the distribution of rotational velocities after the braking mechanism has been applied.

Needless to say, this statistical model is very idealized and represents an extreme case. Most likely h_1 is not a constant even within a spectral sub-type but increases with the original value of h. However, since the observed distribution is not well determined, we do not see any advantage at this moment to build an elaborate model which usually would involve many adjustable parameters. On the other hand, because of this oversimplified model, the quantitative results obtained in this paper represent only a very rough estimate.

If we accept this simplified model, the distribution of equatorial rotational velocities after braking has taken its toll becomes

$$f(x, x_1) = \psi_0(x_1)\,\delta(x-0) + \frac{4}{\sqrt{\pi}}(x+x_1)^2 e^{-(x+x_1)^2} \tag{3}$$

where

$$\psi_0(x) = \frac{2}{\sqrt{\pi}}\left(\int_0^x e^{-t^2}dt - x e^{-x^2}\right) \tag{4}$$

and $\delta(x-0)$ denotes the delta function.

If $\varphi(y, x_1)$ denotes the distribution of observed velocities, y, we have (Kuiper 1935; Chandrasekhar and Münch 1950)

$$\varphi(y, x_1) = y\int_y^\infty \frac{f(x, x_1)\,dx}{x\,(x^2-y^2)^{1/2}} \tag{5}$$

Substituting equation (3) into equation (5) we obtain

$$\varphi(y, x_1) = \psi_0(x_1)\,\delta(y-0) + \psi(y, x_1) \tag{6}$$

where

$$\psi(y, x_1) = \frac{4}{\sqrt{\pi}}\int_0^{\pi/2}(y\sec\theta + x_1)^2 e^{-(y\sec\theta+x_1)^2}d\theta \tag{7}$$

The observed distributions are given in the form of histograms, i.e.,

$$\Phi(y, x_1) = \int_{(n-1)\Delta y}^{n\Delta y} \varphi(y, x_1)\, dy, \qquad (8)$$

$$(n-1)\Delta y < y < n\Delta y.$$

In Figure 1 we have shown 11 computed histograms of $\Phi(y, x_1)$ according to equations (5)-(7) for 11 values of x_1 (0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 1.0, 1.4, 1.8, 2.6) with $\Delta y = 0.2$. This sequence of histograms illustrates the effect of the braking strength on the distribution of observed rotational velocities of stars in a spectral type. The case $x_1 = 0$ corresponds to that of no braking, while the case $x_1 = 2.6$ leads practically to a complete stop of axial rotation by braking.

These diagrams should be compared with the observed histograms. Unfortunately, as we can see in our previous paper (Huang 1953), the two sets of observed diagrams derived from two uncombinable sets of observational data show slight differences. The two uncombinable sets of observational data are respectively from the three-prism and two-prism spectrograms that have been taken during the years by astronomers at the Lick Observatory. There are 1103 stars for which rotational velocities were measured from three-prism spectrograms and 445 stars for which rotational velocities were measured from two-prism spectrograms. Therefore, the histograms of rotational velocities derived from the three-prism spectrograms are definitely more reliable than those from the two-prism spectrograms from a consideration of both the dispersion and the number of stars studied. For this reason we shall compare the computed histograms in Figure 1 with the observed ones obtained from three-prism histograms, the latter being reproduced here in Figure 2. Finally, it may be noted that there are available better observational data for rotational velocities (Slettebak 1949 et seq.; [illegible]

We have mentioned before that by assuming $x_1 \to \infty$ we introduce an approximation. While this approximation does not seriously modify the distribution function at the lower end of observed rotational velocities, it makes an appreciable difference at the upper end. Thus, when we compare histograms (those of $x_1 = 0.3$, 0.4 and 0.8) in Figure 1 with those in Figure 2, we can immediately see a systematic difference arising from this approximation, since in all three histograms obtained from observed data, the distribution function drops much more rapidly than what have been calculated on the assumption of $x_1 \to \infty$. Thus, a cut-off of rotational velocities at their upper end is evident in all three cases in Figure 2. This shows that rotational instability has played a role not only in O- and B-type stars but also in F- and perhaps even later-type stars sometime during the course of their evolution. This result may be significant to our search of clues for star formation.

The strength of braking varies with the moment of braking force and the duration in which braking acts. If indeed the braking is due to magnetic activities which are in turn induced by convection and rotation itself in the early phase of the star's evolution as Schatzman (1962) has suggested, we would expect from the present study that magnetic activities have occurred in stars of all masses in their early phases of evolution, differing only in the strength and duration of such activities. This conclusion is in agreement with Pvoeda's (1964) interpretation of flare stars based on Hayashi's (1961) results of evolution on one hand and Fowler, Greenstein and Hoyle's (1962) theory of formation of light elements--Li, Be, B -- in the early phase of the solar system on the other.

Thus far, we have followed the reasoning of both Schatzman and Poveda that the magnetic field is generated in the star as a result of convective motion and differential rotation. However, it may be noted that it would not modify our conclusion if we assume that the strong magnetic field is derived from accretion of interstellar magnetic lines of force during the period of star formation. If so, the strong magnetic here. However, our crude measurements of stellar rotational velocities provide the only uniform set of data which cover such a large number of stars that make the histograms in Figure 2 statistically significant.

In Table 1 we give the results of our determination of x_1 and v_m for each of three observed histograms after a comparison with those in Figure 1, x_1 being determined by the shape and v_m by the scales of abscissas of both observed and computed diagrams. It should once more be emphasized that these values are very tentative as both theory and observational data (especially the conversion of measured widths of the spectral line to rotational velocities) are crude. However, there is no doubt at all that x_1 is least (but definitely non-vanishing) for O and B- type stars, increases only slightly for A-type stars and becomes much larger for FO-F5 stars. Since few single stars of spectral types later than F5 are found to be rotating with a measurable velocity, we must conclude that x_1 for these late- type stars is greater than 2.6.

While we cannot determine x_1 accurately because of the uncertainty in observational data, the general behavior does indicate that our simple statistical model for the braking effect represents very well the gradual change of the distribution of observed rotational velocities from O and B through A and FO-F5, and finally to later-type stars. Especially it explains the long puzzled fact that a large number of O, B, A stars do not rotate appreciably, although their average rotational velocities as group are high.

The previous result induces us to conclude that contrary to our previous belief, braking is not limited only to stars of spectral types later than F5 but extends to O, B, A and early F stars as well. It suggests that along the main sequence all stars show evidence of having been braked. However, the braking mechanism is weak at the upper branch of the main sequence and increases its strength rapidly after FO. It becomes so strong after F5 that rotation of all stars is practically stopped by it.

activities in the early phase would represent the dissipation because of convection, of magnetic energy that is already there.

II. PROBABILITY OF FORMATION OF PLANETARY SYSTEMS AROUND STARS

It has often been suggested (e.g. Huang 1959) that the disappearance of rotation after F5 may indicate the emergence of planetary systems because it is difficult to understand how the original distribution of angular momenta of the late-type stars should be so radically different from that of early ones. This is especially true if we follow the reasoning in the previous section where the original distribution of angular momenta per unit mass of stars is assumed to follow the Maxwellian distribution (which may be truncated at the high-value end because of rotational instability). Indeed, if angular momenta that have been dissipated from the stars remain in their neighborhood, formation of planetary systems would appear inevitable. Following this line of reasoning we shall make in this section an estimate, though a very crude one, not only of the probability among single stars that will possess planetary systems but also of their general behavior with respect to the spectral type of their parent stars.

Before we shall understand why our planetary system was formed in a state as it is, we are not expected to derive, simply from a consideration of the angular momentum, the detailed structure of planetary systems around stars. In order to obtain a general behavior of any planetary system around a star without going into the detail structure we may define the concept of an "equivalent planet", which is a fictitious planet that would be moving in a circular orbit with an angular momentum per unit mass, h', equal to the average value of the entire mass outside of, but belonging to, the star. Thus, in the case of our solar system, we have (Allen 1955) the total angular momentum of the planetary system equal to 3.15×10^{50} gm cm^2/sec , while its total mass is 2.68×10^{30} gm.

Consequently, its h', denoted hereafter by h'_s, is

$$h_s = 1.176 \times 10^{20}\ \mathrm{cm}^2/\mathrm{sec}. \qquad (9)$$

Then it is easy to find that the equivalent planet of our own planetary system is located at a distance

$$a_s = 1.042 \times 10^{14}\ \mathrm{cm} \qquad (10)$$

from the sun (i.e. between Jupiter and Saturn as would be expected) and revolves with a linear velocity

$$V_s = 11.3\ \mathrm{km}/\mathrm{sec}. \qquad (11)$$

around the sun. It is obvious from our definition of the equivalent planet that it does give us a measure of the extent of any planetary system although it conveys no idea about its total mass and mass distribution in the system.

Let us now assume that the gaseous remnant around a star after its formation is m and the mass, radius and radius of gyration of the star itself are respectively M, R, and Rk. Therefore, the total angular momentum of the star is

$$\Omega(v) = M R^2 k^2 v \qquad (12)$$

After braking, all of its angular momentum is transported into the surrounding medium (of mass m) according to our theory if $v \le v_1$ $(= x_1 v_m)$ and a constant part (equal to $\Omega_1 = M R^2 k^2 v_m x_1$ of it is transported out if $v > v_1$. However, we should realize that the angular momentum in the nebula that will evolve to become a planetary system could be different from what has been fed into it by the star, because some angular momentum, say Ω_0, may be originally associated with it and has never gone into the star. Since both the star and its surrounding nebula are supposed to have been formed from the same interstellar medium, we may reasonably assume that the vector, $\vec{\Omega}_0$, points to the same direction as the stellar angular momentum does. Hence, we can consider their magnitudes only and the angular momentum associated with the nebula (of mass m) after the completion of braking process has the following value:

$$\Omega(x) = MRk^2 v_m x + \Omega_0 \quad \text{for stars with } x \leq x_1$$
$$= MRk^2 v_m x_1 + \Omega_0 \equiv \Omega_1 \quad \text{for stars with } x > x_1 \tag{13}$$

For stars with slow rotation which indicates smallness of h of the pre-stellar medium, Ω_0 must be small and may be neglected. However, for stars of rapid rotation and especially for those which have passed through the stage of rotational instability in the course of their evolution, h of the prestellar medium must be large and Ω_0 large too.

If we include the possibility that a part of the angular momentum in the nebula may be dissipated away (for example through the loss of mass), Ω_0 may be even negative. Thus, Ω_0 cannot be estimated from the present consideration. However, as we shall see, this uncertainty can be formally circumvented in the following calculation.

Since

$$h' = \frac{\Omega}{m} = (GMa)^{1/2}, \tag{14}$$

where a is the radius of the equivalent planet's orbit around a star and

$$h'_s = a_s V_s = (GM_\odot a_s)^{1/2} \tag{15}$$

for the planetary system of the sun, we may introduce a new variable

$$\xi = \left(\frac{Ma}{M_\odot a_s}\right)^{1/2} \tag{16}$$

to measure the size of the equivalent planet's orbit around any star. It follows from equations (13)-(16) that ξ extends from

$$\xi_0 = \frac{\Omega_0}{m a_s V_s} \quad \text{to} \quad \xi_1 = \frac{\Omega_1}{m a_s V_s} \tag{17}$$

if Ω_0 is positive and zero to ξ_1, if Ω_0 is negative. Therefore, the lower and upper limit of the radius of the equivalent planet's orbit denoted respectively by a_0 and a_1 are given by

$$\xi_0 = \left(\frac{M a_0}{M_\odot a_s}\right)^{1/2} \quad \text{and} \quad \xi_1 = \left(\frac{M a_1}{M_\odot a_s}\right)^{1/2} \tag{18}$$

Now x and ξ are related by

$$x = \lambda \xi - \frac{\Omega_0}{M R k^2 v_m} \tag{19}$$

where

$$\lambda = \frac{m}{M} \frac{a_s}{R} \frac{V_s}{v_m} \frac{1}{k^2} \tag{20}$$

It is easy to derive from equations (1) and (19) that the distribution function $g(\xi)$ with respect to ξ.

If we further let

$$\ell = \xi - \frac{\Omega_0}{m a_s V_s} \tag{21}$$

and

$$t_1 = \xi_1 - \frac{\Omega_0}{m a_s V_s} = \frac{x_1}{\lambda} \qquad (22)$$

we obtain from $S(\xi)$ that:

$$g\left(t + \frac{\Omega_0}{m a_s V_s}\right) = \frac{4}{\sqrt{\pi}} \lambda^3 t^2 e^{-\lambda^2 t^2} + \delta(t - t_1)\left[1 - \phi(x_1)\right] . \qquad (23)$$

Since from t to ξ it involves only a horizontal translation, the shape of the distribution function does not depend upon Ω_0. If Ω_0 is positive, we simply translate the entire curve to the right, thereby increasing systematically the size of the expected orbits. If Ω_0 is negative, we translate the curve to the left and set the probability to zero everywhere that is on the left side of the origin. In this way we are able formally to take into account any value of Ω_0 . Physically, however, we should remember that Ω_0 likely varies from case to case and moreover we do not know how it does vary (say, with respect to M, h, m, h', etc.). Hence, it is one of the uncertain factors that prevent us to derive the probability of occurrence of a planetary system of a given size.

Since we can obtain easily the distribution with respect to ξ from that with respect to t , we shall consider hereafter only the latter which, as we see from equation (24), depends upon two parameters, λ and t_1, or equivalently λ and x_1 because of equation (23). While x_1 has been determined from observed data, t_1 involves an unknown parameter, namely m/M.

From the values of x_1 and v_m in Table 1 we can compute t_1 (or λ) for each of the three cases given there. We shall use the mean radius $R/R_\odot = 3.9$ and mean mass $M/M_\odot \doteq 5.9$ for O and B stars, $R/R_\odot = 1.74$ and $M/M_\odot = 2$ for A stars and $R/R_\odot = 1.35$ and $M/M_\odot = 1.4$ for F0-F5 stars. In all cases we adapt $k^2 = 0.05$. Then it follows from Table 1 and equations (20) and (23) that

mt_1/M is equal to 6.9×10^{-4} for O and B stars, equal to 4.1×10^{-4} for A stars, and equal to 6.4×10^{-4} for F0-F5 stars. It is quite evident that the error involved in the determination of x_1 and ν_m is much greater than the differences we have found in these three expressions of t_1. Consequently, we may roughly set

$$t_1 = 6 \times 10^{-4} \frac{M}{m} \qquad (24)$$

as representing all stars from O to F5. If $\quad = 0$, equation (24), when combined with equations (17), (18) and (23), gives

$$\frac{a_1}{a_s} = (6 \times 10^{-4})^2 \frac{M M_\odot}{m^2} \qquad (25)$$

which states that the maximum size of the planetary system is directly proportional to the mass of its parent star and inversely proportional to the square of its total mass. It should be noted that the numerical coefficient in equation (24) does not apply to stars later than F5 for which we do not have any observed data for making the calculation.

We have seen how critically does the size of a planetary system depend upon the mass in the nebula. A further complication is the dissipation of mass in the nebula and accompanying dissipation of angular momentum before the formation of a planetary system of finite bodies. The difficulty of this point can be seen from the fact that even in our own planetary system there is no consensus concerning the amount of mass in the solar nebula in the beginning, as some investigators advocate a value of m/M = 0.1 while others use considerably smaller values.

Since the actual size of any planetary system must be equal to or less than the upper limit, we have for our own planetary system $a_1 \geqslant a_s$. It then follows from equations (1.) and (18) that

$$\frac{m}{M_\odot} \leq \frac{\Omega_0}{a_s V_s M_\odot} + \left(\frac{R_\odot}{a_s}\frac{v_m}{V_s}\frac{\ell^2}{R}\right)x_1 \qquad (26)$$

The ratio m/M for the solar system at present is equal to 1.3×10^{-3}. In the early days it must be much greater than this value. If we now take v_m to be the same as that determined in Table 1 (200 Km/sec), we have

$$\frac{R_\odot}{a_s}\frac{v_m}{V_s}\frac{\ell^2}{R} = 5.9 \times 10^{-4} \qquad (27)$$

Therefore, it follows from equation (26) that if $\Omega_0 = 0$, $x_1 > 2.2$. Equation (1) shows that the probability of having $x > 2.2$ is rather small. This would put our planetary system as an unusual case, namely its chance of occurrence is small. The other alternative is that Ω_0 is appreciable. This means that a significant portion of the angular momentum that is now found in our planetary system has never belong to the sun. The present consideration cannot decide which alternative is the more plausible one.

Thus, we have seen many uncertainties concerning planetary systems around the stars in general. They cannot be resolved for the time being. What we can predict from the present simple theory is that the size of a planetary system increases with a decrease of its total mass and that the distribution of sizes (measured by ξ of the equivalent planet's orbit) behaves like a Maxwellian distribution of velocity magnitudes at the lower end and is truncated at the upper end. For O, B and A stars the truncation occurs at relatively small values. Therefore unless Ω_0 has a large variation among individual systems, their sizes do not vary greatly from one to the other. Most of them are crowded at the upper limit. The truncation of systems around F-type stars occurs at relatively larger values. Consequently, the spread of sizes is also larger, although there is still an appreciable number of them at the upper limit of their size. On the other hand, the sizes of planetary systems around late-type (later than F) stars are expected to be distributed like a complete Maxwellian curve with a wide spread of values. They should not show the tendency to accumulate at any particular size like systems around early-type stars do. All these consequences would be modified if Ω_0 varies greatly from one system to another; in such cases the distribution of planetary sizes can be derived only when we know the variation of Ω_0.

Finally, we would like to ask if it is possible that there is no mass left after the actual processes of star formation (i.e., m = 0). In that case no planet would be formed. Indeed from the point of view of star formation we do not have any compelling reason to assure us that there must be some mass left behind to form planets after the star itself is formed. On the other hand, the rotational behavior of stars does suggest that there must be some mass left behind since axial rotation could not be effectively braked without the presence of such mass around the star, although we cannot at present estimate the amount of mass that is there. It is therefore the main point of this paper, if nothing else, to show the behavior of axial rotation of stars indicates the high probability of occurrence of planetary systems around stars

The present paper perhaps represents the first serious attempt to link the problem of occurrence of planetary systems in the stellar universe with some observational facts. The uncertainties in both theory and observation force us to adapt an over-simplified model. Thus, it should be regarded as a pioneer exploration of a nebulous field of learning rather than a legitimate treatment of something that is clearly understood. As a result we emphasize only the qualitative conclusions that have been derived here.

It is my pleasure to express my thanks to Mr. Clarence Wade, Jr. for the numerical integrals in equations (7) and (8) on the IBM 7094 digital computer at Goddard.

REFERENCES

Allen, C. W. 1955, Astrophysical Quantities (London: Athlone Press).

Chandrasekhar, S. and Münch, G. 1950, Ap. J. 111, 142.

Edgeworth, K. E. 1946, M.N. 106, 470, 476 and 484.

Fowler, W. A., Greenstein, J. L. and Hoyle, F. 1962, Geophys. J. RAS. 6, 148 .

Hayashi, C. 1961, Pub. Astr. Soc. Japan, 13, 450.

Hoyle, F. 1960, Quarterly J.RAS. 1, 28.

Huang, S.-S. 1953, Ap. J. 118, 285.

Huang, S.-S. 1959, P.A.S.P. 71, 421.

Huang, S.-S. and Struve, O. 1954, Ann. d'Ap. 17, 85.

Huang, S.-S. and Struve, O. 1960, "Stellar Atmospheres" Ed. J. L. Greenstein (Chicago, Chicago University Press) Chap. 8. p. 321.

Kuiper, G. P. 1935, P.A.S.P. 47, 15 and 121.

Poveda, A. 1954, Nature 202, 1319.

Schatzman, E. 1962, Ann. d'Ap. 25, 18.

Slettebak, A. 1949, Ap. J. 110, 498.

Struve, O. 1930, Ap. J. 72, 1.

Struve, O. 1945, Pop. Astr. 53, 201 and 259.

TABLE I

Values of x_1 and v_m determined from a Comparison of Observed and Calculated Histograms of Rotational Velocities

Sp. Type	x_1	v_m(km/sec)
O and B	0.3	~200
A	0.4	~200
F0-F5	0.8	~200
later than F5	> 2.6	---

LEGENDS OF FIGURES

Figure 1 - Computed histograms of projected rotational velocities of groups of stars that have suffered different degrees of braking. We have plotted here 11 histograms for 11 values of χ_1 which, according to our model, measures the strength of braking of stellar axial rotation.

Figure 2 - Observed histograms of projected rotational velocities of three groups of stars, O and B, A, and F0-F5, taken from a previous paper. The histogram of projected rotational velocities for later type stars will be consisted of only a single column of small velocities. These histograms should be compared with those in Figure 1. The trend of increasing braking strengths from O and B, through A and F0-F5 to later-type stars is unmistakable. From the comparison we may make a rough estimate of χ_1 for each of these three groups of stars.